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(6) BLAST OVERVIEW AND NEAR-FIELD EFFECTS

by

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This report covers material presented at a workshop on high explosives incendiary modeling and was intended to set the stage for presentations to follow on the effects of multiple fragments and their effects on aircraft components.

In this setting, blast is described as a ubiquitous damage agent associated with many conventional and unconventional (FAE and nuclear) weapons. The damage produced in an aircraft target by a high explosive incendiary projectile is generally quite localized in its effect. The effects are limited to a zone quite close to the projectile at its time of detonation, and the damages are produced by the combined action of the fragments from the projectile, the products of explosion, and also from the air blast produced.

Two convenient divisions of blast are classified as internal and external. Reference to internal blast is when the damage is applied inside the structure of interest, and external blast is when the damage agent is applied to the exterior surface of the structure of interest.

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INTRODUCTION

This report covers material presented at a workshop on high explosives incendiary modeling and was intended to set the stage for presentations to follow on the effects of multiple fragments and their effects on aircraft components.

In this setting, blast is described as a ubiquitous damage agent associated with many conventional and unconventional (FAE and nuclear) weapons. The damage produced in an aircraft target by a high explosive incendiary projectile is generally quite localized in its effect. The effects are limited to a zone quite close to the projectile at its time of detonation, and the damages are produced by the combined action of the fragments from the projectile, the products of explosion, and also from the air blast produced.

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EXTERNAL BLAST

External blast is probably the least efficient damage agent in current study by the warhead community since, in the majority of the blast field, around 99.9% of the incident energy in the blast wave is reflected from the target and only 0.1% is useful in damaging the target. The saving graces are that, for many weapons, there is so much of it and it does tend toward an isotropic damage agent.

Kinney's report[1] gives an excellent overview of the structure of composite blast waves. Figures 1 through 6 are taken from this report and will serve as a means of defining the near-field for a bare charge.

Figure 1 is a relatively complicated pressure structure for a composite blast wave. Figure 2 shows density, particle velocity, and temperature profiles at a distance of 5 charge radii from explosion center.

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[1] Naval Weapons Center. *Engineering Elements of Explosions*, by G. F. Kinney. China Lake, Calif., NWC, November 1968. 52 pp. (NWC TP 4654, publication UNCLASSIFIED.)

Figure 3 is a pressure profile for a composite blast reaching out to 11 charge radii, is extended out to 25 charge radii in Figure 4, and still further to 50 charge radii in Figure 5. Figure 6 is a summary, in log-log coordinates of the pressure-distance relations for air-shock front, the products-air interface, and for the direct pressures of the expanding products cloud.

External blast damage is treated in a number of ways. The current criteria in use in the community are summarized in Table 1. The cube root scaling is essentially a peak pressure criterion; the BRL (O. T. Johnson) scaling is midway between a peak pressure criteria and a reflected impulse criterion; the British square root scaling is midway between a peak pressure and a side-on impulse criterion; and the critical impulse in a critical time criterion produces criteria that shift over the full range from peak pressure to side-on impulse, depending on the target and the charge size. The author is personally prejudiced toward the later criterion.[2]

TABLE 1. Blast Damage Radius Criteria in Use in Lethality Community.

Criterion	Basis
$R = KW^{1/3}$	Peak Pressure
$R = KW^{0.435}$	Least Squares Fit
$R = KW^{1/2}$	Energy Flux
$R = KW^{0.55}$	Reflected Impulse
$R = KW^{2/3}$	Side-On Impulse
$R = KW^{\eta}$, $1/3 \leq \eta \leq 2/3$	Critical Impulse in Critical Time (Hyperbola on pressure-impulse plane. Lehigh, 1946)

FRAGMENT CASE IMPULSE

If we assume that the quantity of major interest to us is the reflected impulse, and if we further simplify the problem by assuming that

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[2] Robert G. S. Sewell and G. F. Kinney. "Response of Structures to Blast, A New Criterion," *Annals of the New York Academy of Sciences*, Vol. 152, Article 1 (October 28, 1968), pp. 532-47.

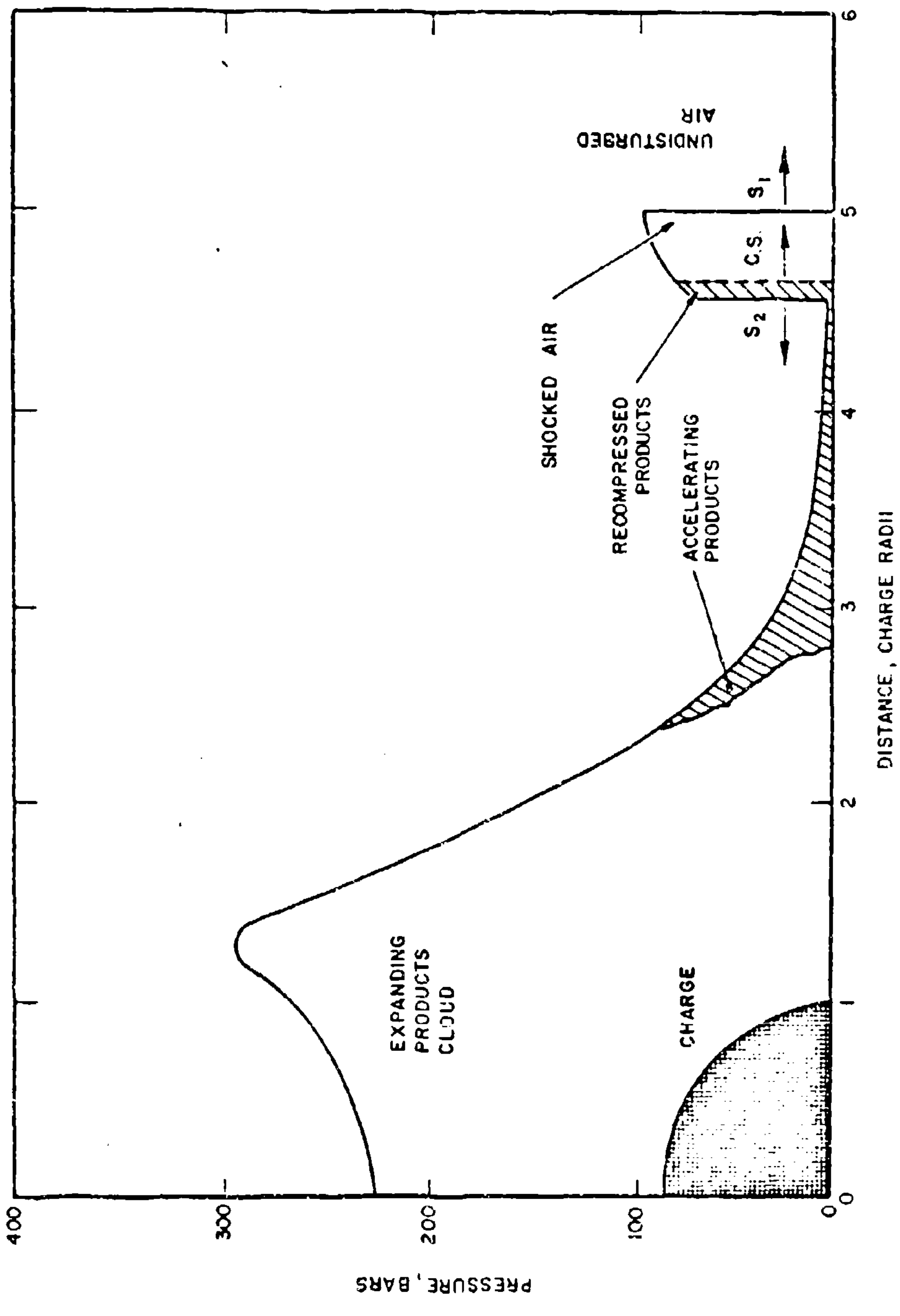

FIGURE 1. Pressure Profile When Shock Front is at 5 Charge Radii.

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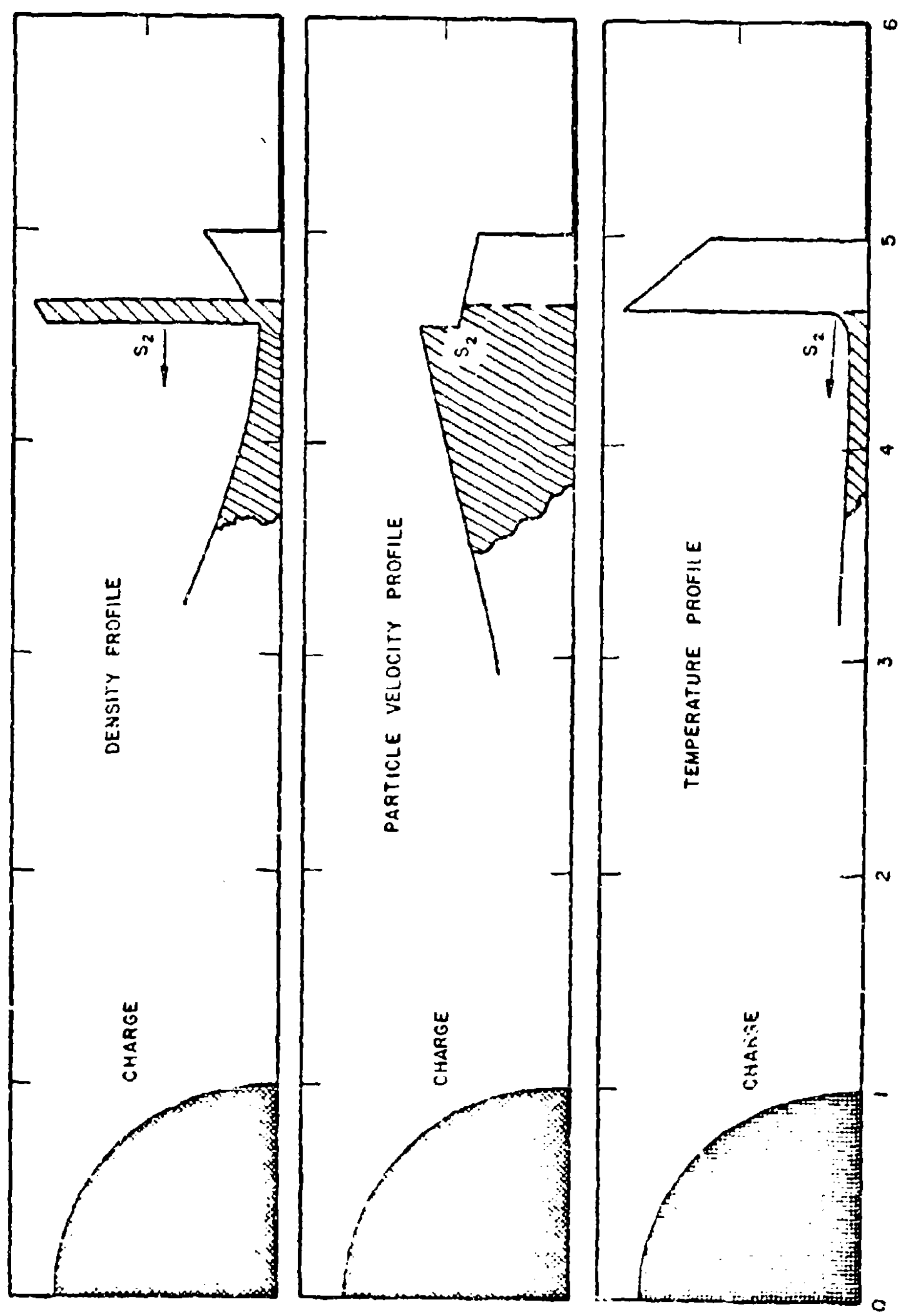

FIGURE 2. Density, Particle Velocity, and Temperature Profiles.

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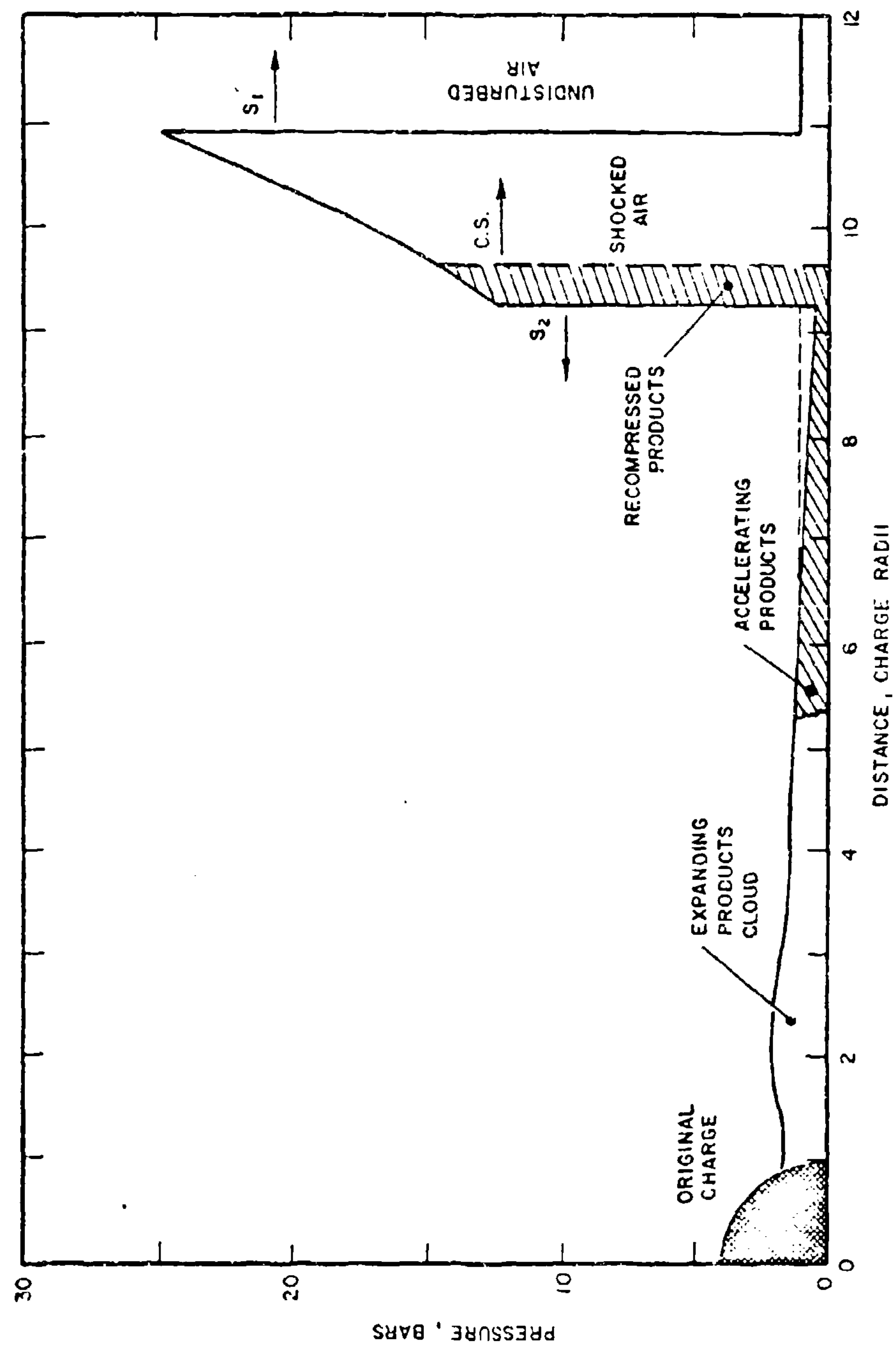


FIGURE 3. Pressure Profile When Shock Front is at 11 Charge Radii.

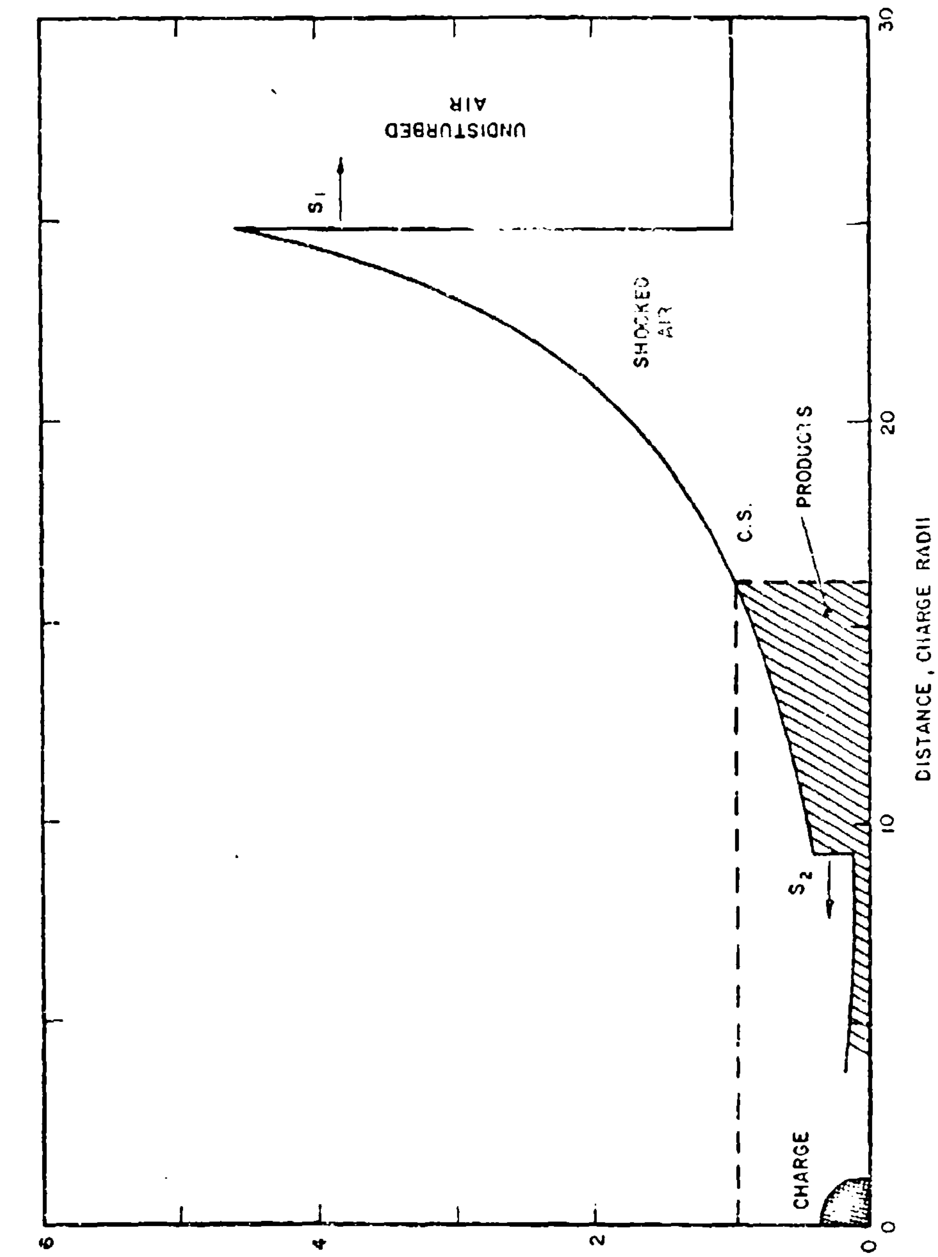


FIGURE 4. Pressure Profile for Shock Front at 25 Charge Radii.

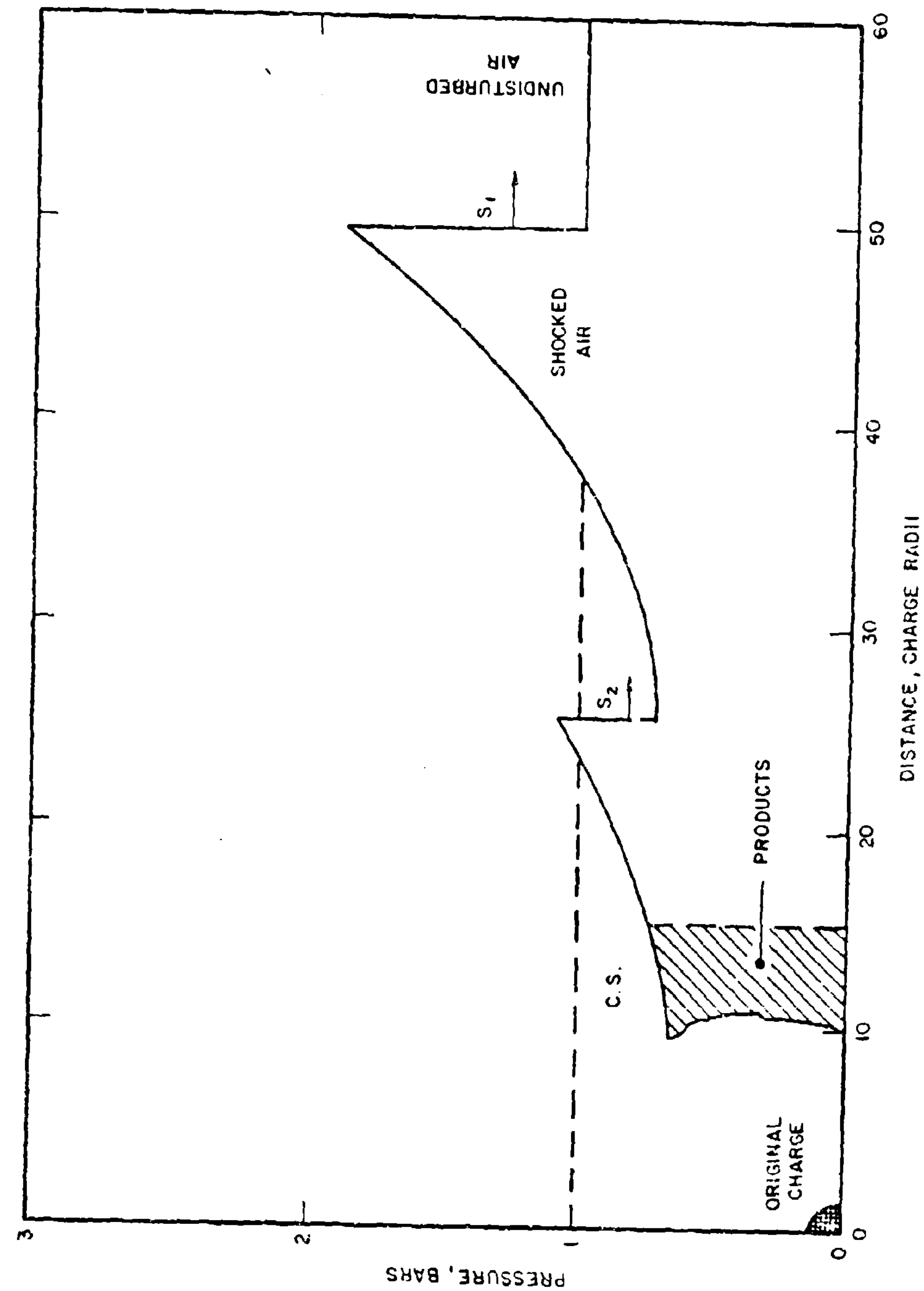


FIGURE 5. Pressure Profile for Shock Front at 50 Charge Radii.

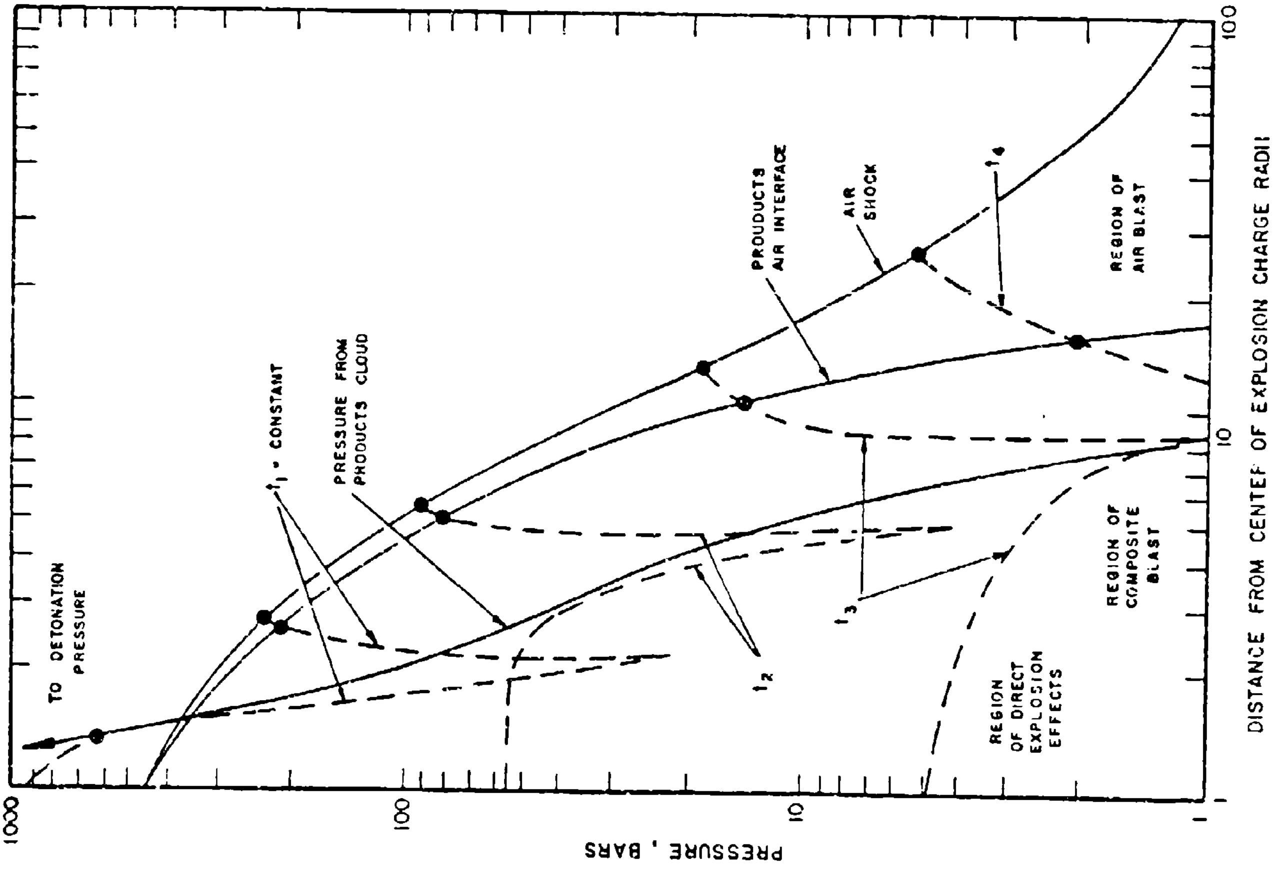


FIGURE 6. Pressure-Distance-Time Relations for Reference TNT Explosion.

the target will not be perforated by the fragments (a practical situation for 20 mm HEI fragments against 1/4-inch aluminum sheet), we can calculate the momentum per unit area in the fragment system and compare it to the reflected impulse from a bare explosive charge. To enhance the calculation in favor of the blast, the full explosive weight will be used with no Fano correction. Spherical geometry is assumed.

For example, let us examine a device with a charge-to-metal ratio of 0.33, a charge of 6 grams with a density of 1.65 grams/cm^3, and a steel case of 20 grams with a density of 7.8 grams/cm^3. The comparison of the impulse per unit area of the fragments and the reflected impulse from the blast are shown in Figure 7.

If the device examined had been moving at 1,000 m/sec, the additional effects in the forward direction are easily estimated for the fragments but not for the blast. The blast is influenced by the pressure distribution around the charge and the velocity gradient in the products cloud. While linear momentum is still conserved, the system in which conservation applies includes the air that has been influenced by the shock wave. A simplified method of treating the moving charge blast is to compute a shifted burst point and treat the blast as spherical from that point. A more general approach is shown in Appendix A. In a vacuum, the center of mass continues in its original state of motion, but in reasonable atmospheres the blast center slows rapidly. Figure 8 shows a plot of suggested shifts in burst point as a function of velocity with altitude as a parameter. The kinetic energy of the explosive charge should be added to the charge weight. A convenient formula for such a correction is shown in equation (1).

$$W_e = W\left(1 + \frac{V^2}{9}\right) \qquad (1)$$

where V is in km/sec

INTERNAL BLAST

Confined internal blast is the most efficient part of blast as a damage agent. In an underwater explosion, approximately 45% of the explosive energy appears as kinetic energy of the surrounding water at the first bubble maximum. In a cased explosive charge, 25 to 35% of the detonation energy appears as kinetic energy of the fragments.

Detonation of an explosive charge such as TNT or H-6 in an air-filled unvented chamber is still more efficient on a weight basis, as these explosives are oxygen-deficient, and burning of the excess fuel in the air's oxygen makes available the heat of combustion rather than the heat of detonation.

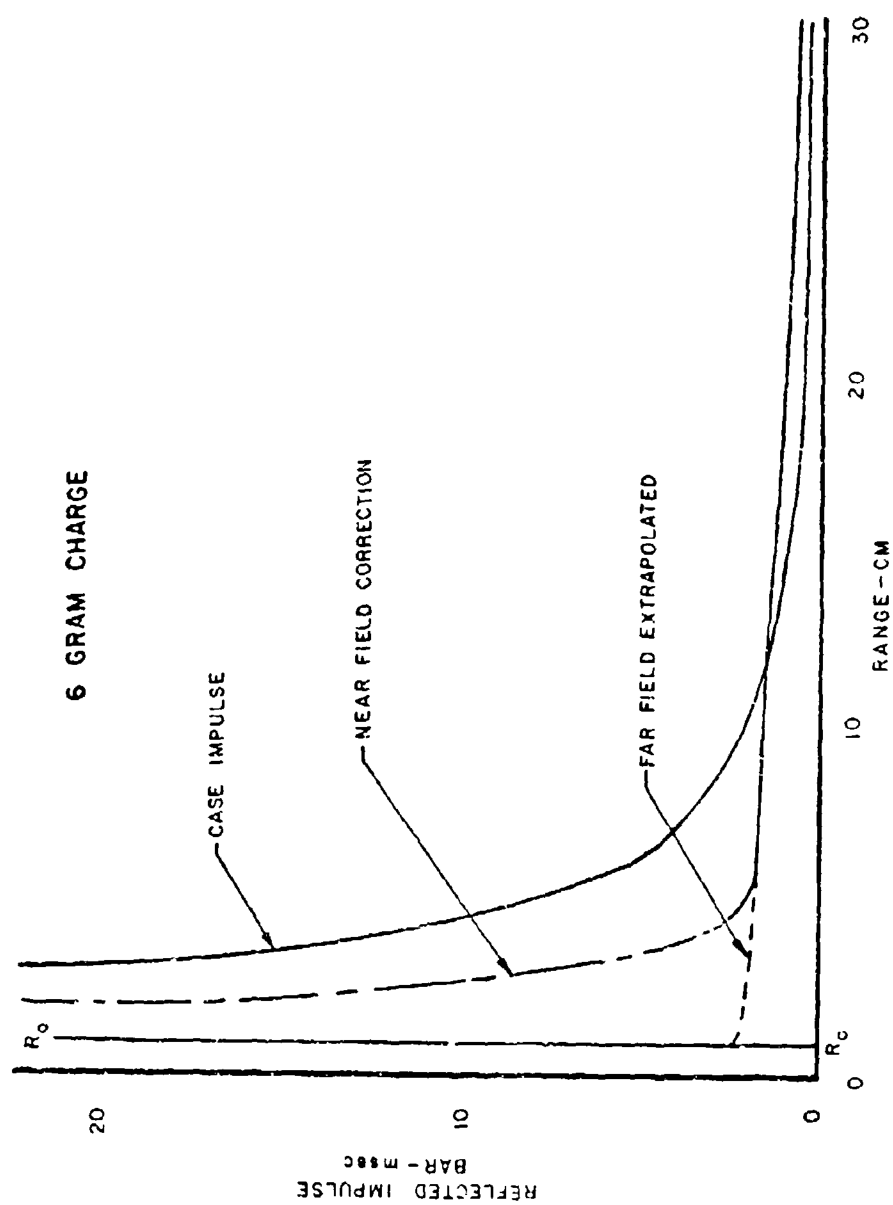


FIGURE 7. Comparison of Air Blast Reflected Impulse With and Without the Near Field Correction with Case Fragment Impulse.

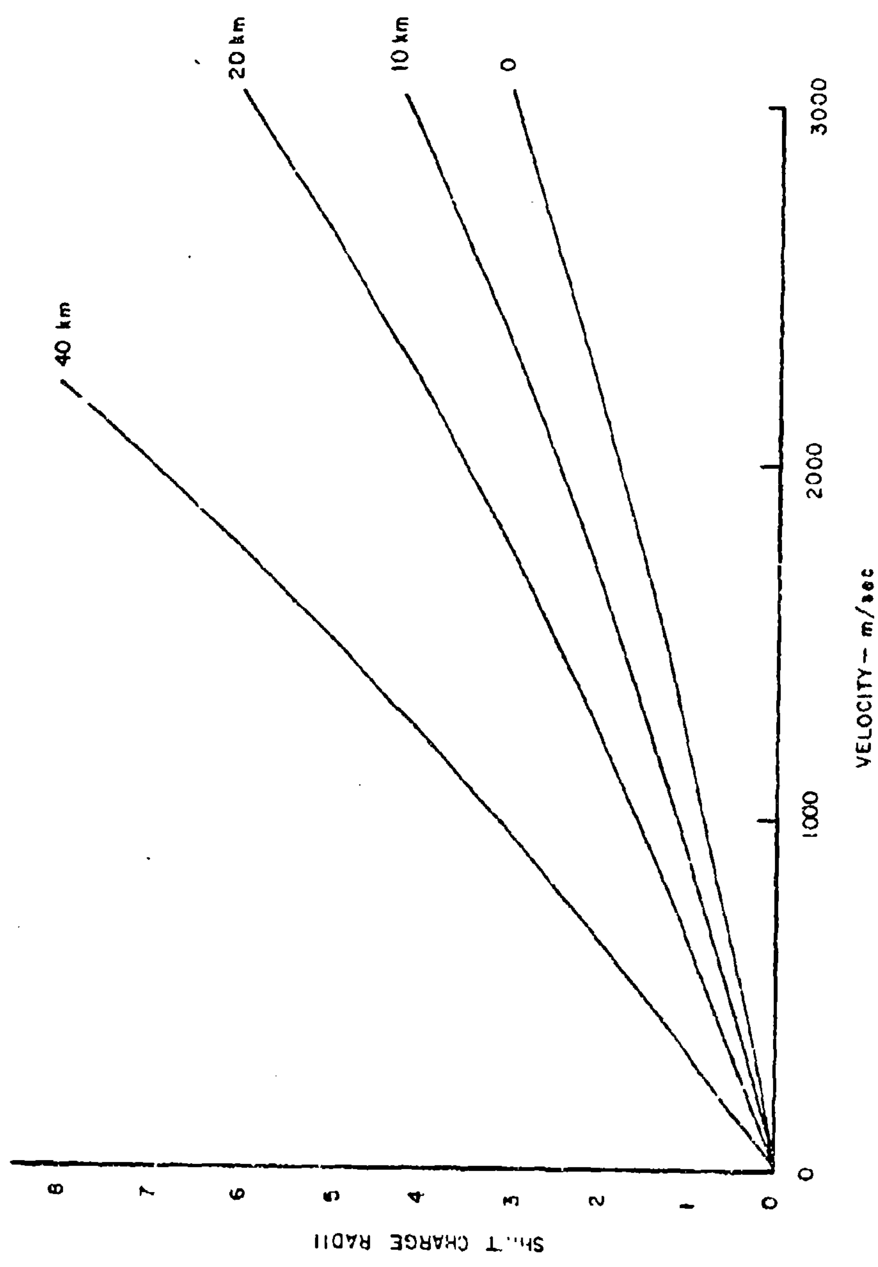


FIGURE 8. Estimated Shift of Center of Explosion for Use with Moving Charges. *Shift in Charge Radii as a Function of Velocity with Altitude as a Parameter.*

The restrictions are important; an unvented chamber and sufficient oxygen. If the chamber is vented above a critical amount based on the vent area to chamber surface area ratio, the combustion effect is removed and even for nearly oxygen-balanced explosives such as PETN, the total impulse on the chamber walls is decreased by a factor of about 20. The critical vent area to surface area ratio for most internal detonations is of the order of 10%. The detonation energy is always available.

Even in an overvented chamber, the wall loading due to the direct blast waves and from the multiple reflected blast waves of the corners is available for damage production[3] (see Figures 9-11).

The detonation of an HEI projectile equipped with a delay fuze inside an aircraft can occur in a variety of environments ranging from being immersed in liquid, surrounded by a variety of components in a larger compartment (too large for compartment wall damage), noncritically vented chambers, to overcritically vented chambers essentially devoid of components.

IN A LIQUID SURROUND

When the detonation occurs in a liquid, the liquid surround becomes a part of the fragment system attacking the container walls. But due to cavitation effects produced by the rapid acceleration of the container walls, the deformation and rupture of the container wall becomes dependent on the energy flux, and the radius of damage is found to be proportional to the square root of the explosive charge. The constant of proportionality should normally depend on the density of the liquid and the density, thickness, and strength of the wall material. However, as is found in underwater attack of ships, a constant value will probably be adequate. The radius within which the container wall would be ruptured is shown in equation (2).

$$R = K_e W^{1/2} \qquad (2)$$

UNITS:

W	K	R
lbs	2 ft/lb$^{1/2}$	ft
kg	0.91 m/kg$^{1/2}$	m
grams	2.86 cm/gram$^{1/2}$	cm
grains	0.287 in/grain$^{1/2}$	in

[3] Civil Engineering Laboratory. *Blast Environment from Fully and Partially Vented Explosions in Cubicles*, by W. A. Keenan and J. E. Tancreta. Port Hueneme, Calif., CEL, November 1975. (Technical Report R828, publication UNCLASSIFIED.)

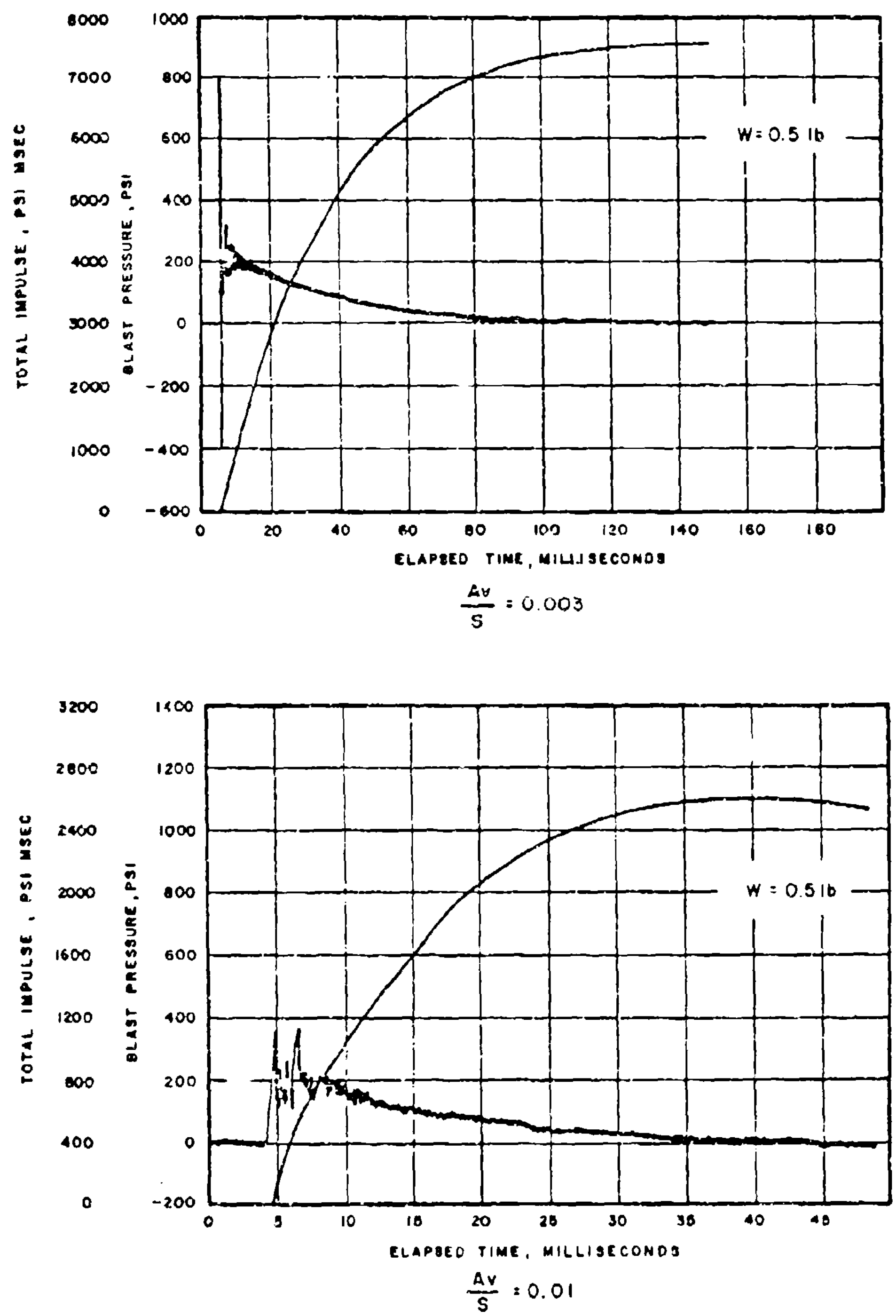


FIGURE 9. Pressure and Impulse as a Function of Time for Partially Vented Cubicles.

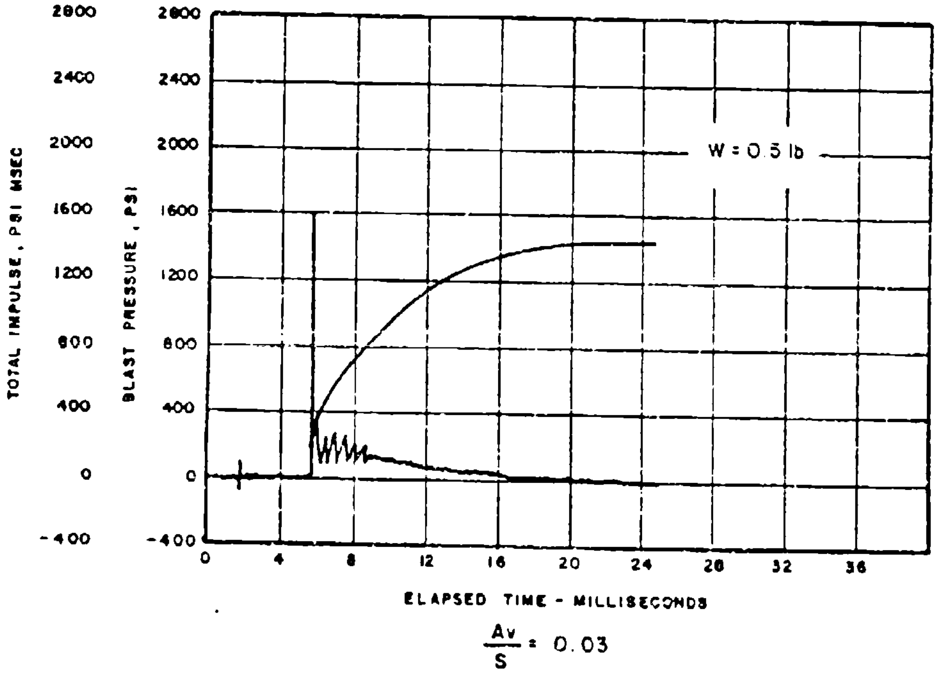
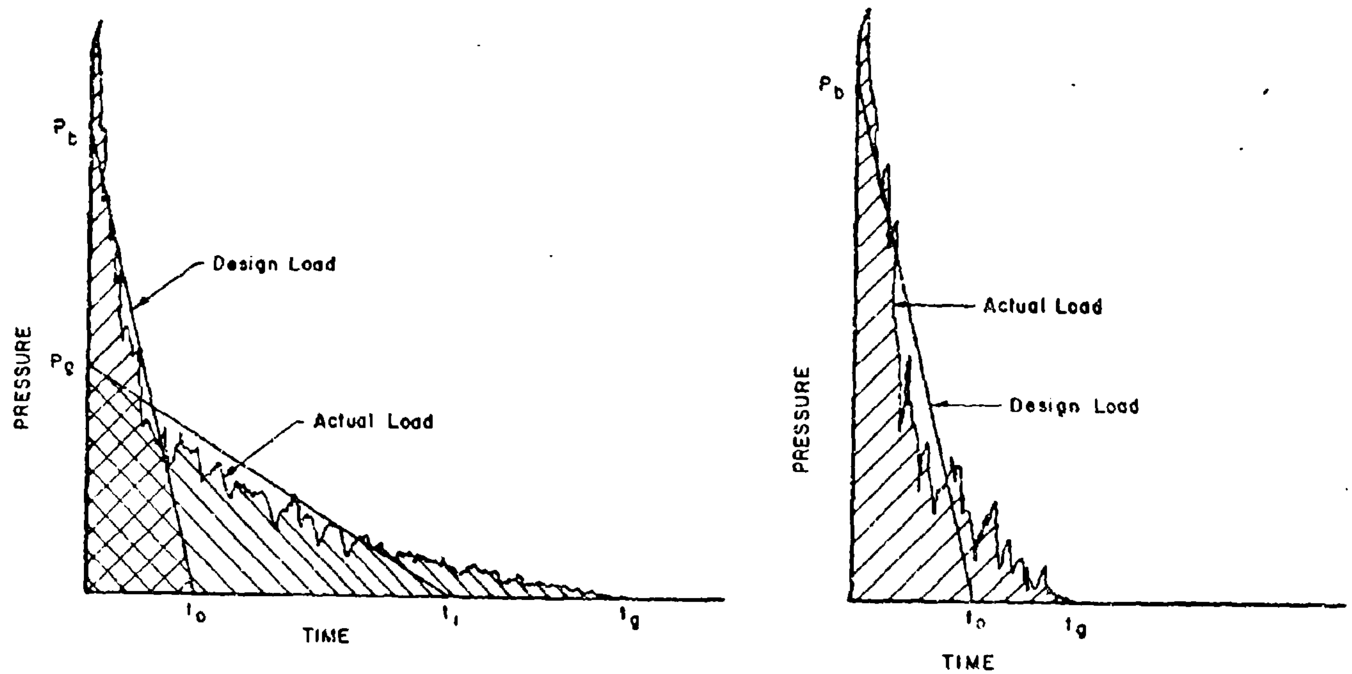


FIGURE 10. Pressure and Impulse as a Function of Time for a Nearly Fully Vented Cubicle.



(a) Inside partially vented cubicle, $A/V^{2/3} \leq 0.60$

$\frac{A_v}{S} \leq 0.10$

(b) Inside fully vented cubicle, $A/V^{2/3} \geq 0.60$

$\frac{A_v}{S} \geq 0.10$

FIGURE 11. Comparison of the Blast in Partially Vented and Fully Vented Compartments.

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NEAR-FIELD EFFECTS

When the detonation occurs in a large compartment containing a large number of components, a zone will be created which will be empty of components (P α 100 kb) surrounded by a compressed zone where the components have been totally or partially disabled by impact during the compression. Using the Gurney formula for predicting the velocity that can be given the surrounding medium when treated as an average density of material, and the unit rupture work[4] as the limit energy that can be absorbed by the support structure (assumed to be aluminum in aircraft structures), the radii can then be calculated by using equation (3) that will describe these zones.

$$R_{NF} = K \left(\frac{1}{\rho_m}\right)^{1/3} W^{1/3} \tag{3}$$

R_{NF} is the threshold limit of the zone of damage, and the radius of the zone of total removal (R_{TNF}) would be about one quarter of the threshold radius.

K is dimensionless, but the density must be expressed in units which match the charge units and the desired length units. For aluminum structure, K = 3.09, and R_{NF} can be calculated by using equation (4).

$$R_{NF} = 3.09 \left(\frac{1}{\rho_m}\right)^{1/3} W^{1/3} \qquad P_k = 0 \tag{4}$$

The radius of total removal can be calculated by using equation (5).

$$R_{TNF} = 0.77 \left(\frac{1}{\rho_m}\right)^{1/3} W^{1/3} \qquad P_k = 1 \tag{5}$$

DIRECT BLAST ON WALL

There are those cases where the detonation occurs in proximity to the skin or a compartment wall in such a geometry that the fragment beam does not impact the wall. However, a direct blast may rupture the skin or wall. The damage mechanism utilizes the reflected impulse, which will be delivered under the conditions of HEI detonations in times short compared to the resonant period of the structures under consideration. A

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[4] Theodore Baumeister, Ed. *Marks' Standard Handbook for Mechanical Engineers*, Seventh Edition. New York, McGraw-Hill, 1967.

geometric limitation is also imposed due to the sharp drop in the reflection coefficient at the critical angle for formation of the Mach stem, 39°58', or for all practical purposes, 40 degrees. Since the reflected impulse scales as $W^{0.55}$, the radius of direct blast (R_{DB}) can be represented by equation (6).

$$R_{DB} = K_{DB}\, W^{0.55} \tag{6}$$

The constant, K_{DB}, is dependent on the density of the wall, the strength of the wall, and the thickness of the wall. These are contained in the critical impulse in the critical time criterion, which for simplicity will be shortened to a critical reflected impulse criterion, and is represented by equation (7).

$$K_{DB} = K_S \frac{V_o \rho_o \delta_o}{V_s \rho_s \delta_s} \tag{7}$$

UNITS	K_S	UNITS OF V, ρ, δ
$m/kg^{0.55}$	2.0	(m/sec, kg/m^3, mm)
$ft/lb^{0.55}$	0.227	(ft/sec, $slugs/ft^3$, in.)

V_o = 91.44 m/sec or 300 ft/sec = critical particle velocity of aluminum
ρ_o = 2.8 x 10^3 kg/m^3 or 5.48 $slug/ft^3$ = density of aluminum
δ_o = 1 mm or 1 inch = reference thickness
$_s$ = subscript for actual structure

CLOSED COMPARTMENT

The reason for treating the direct blast first was that if the direct blast blows a hole in the compartment wall, sufficient to critically vent the chamber, then the gas pressure phase of the internal explosion, which is responsible for the majority of the impulse on the walls, will be eliminated. The gas pressure phase is dependent on several factors; the charge to free volume ratio which, depending on the atmospheric pressure, determines the fuel to oxygen ratio and the pressure rise in the compartment; and the vent area to surface area ratio. Since the loading time is large compared to the resonant period of the structure, the impulse criterion becomes a peak pressure criterion and the radius of effects will vary as the cube root of the explosive.

The proportionality constant is similar to that for the near-field effects. The density of interest does not include the components in the volume, but only the mass of air and the mass of the walls. Again assuming aluminum as the wall material, the damage radius (K_g) is as shown in equation (8).

$$K_g = 3.09 \left(\frac{1}{\rho_m} \frac{V}{V_f}\right)^{1/3} \left(3- 20 \frac{A_v}{S}\right)^{1/3} \left(\frac{P_x}{P_o}\right)^{1/3} \text{ m/kg}^{1/3} \tag{8}$$

$$\frac{P_x}{P_o} = \frac{\text{Compartment pressure}}{\text{Sea level pressure}} = e^{-h/9.69} \tag{9}$$

$$\rho_A = \text{air density} = 1.23 \ e^{-h/9.69} \text{ kg/m}^3 \tag{10}$$

RETURN TO THE NEAR-FIELD

Let us go back now to the attack on a 1/4-inch aluminum plate with first the spherical charge and then with an actual 20 mm HEI projectile with instantaneous fuzing.

The typical characterization of the fragments from the 20 mm HEI projectile gives fragments of such low mass that even with the vector addition of the 1,000 m/sec (3,280 ft/sec), the fragments are below the ballistic limit for the 1/4-inch (6.35 mm) aluminum.

Assuming the same fragment size distribution for the spherical charge, let us examine the momentum delivered to the plate by the fragments, the reflected impulse (air shock) and examine to see if near-field corrections are required. Let us arbitrarily set the burst point at 30 mm (1.18 inch) from the plate.

The critical impulse for the plate is 16.22 - bar - msec. Even without the additional momentum in the forward directed fragment cloud, at 30 mm the central region would deliver 21.2 - bar - msec. This establishes the possibility of producing a plugging failure of at least some of the plate.

With the addition of the incoming velocity, the central region of the fragment cloud will deliver more impulse than would be obtained from first adding the incoming velocity to the fragment velocity for the decreased time to reach the target surface, which means that less expansion has taken place, and that the mass impacting per unit area will also increase. Treating the expanding fragment case as a constant volume shell, the sequence of events and the magnitude of the impulse being delivered can be shown diagrammatically as in Figure 12. In actual fact, the case ruptures before the case has expanded to twice the original size, and explosive products begin to flow between the fragments. The initial flow velocity is of the order of 6.6 mm/μsec (21,648 ft/sec); so during the later phase of the interaction depicted, the beginning stages of the air shock formation and the streaming product cloud will reach the target ahead of the fragments. The central region of the target will receive

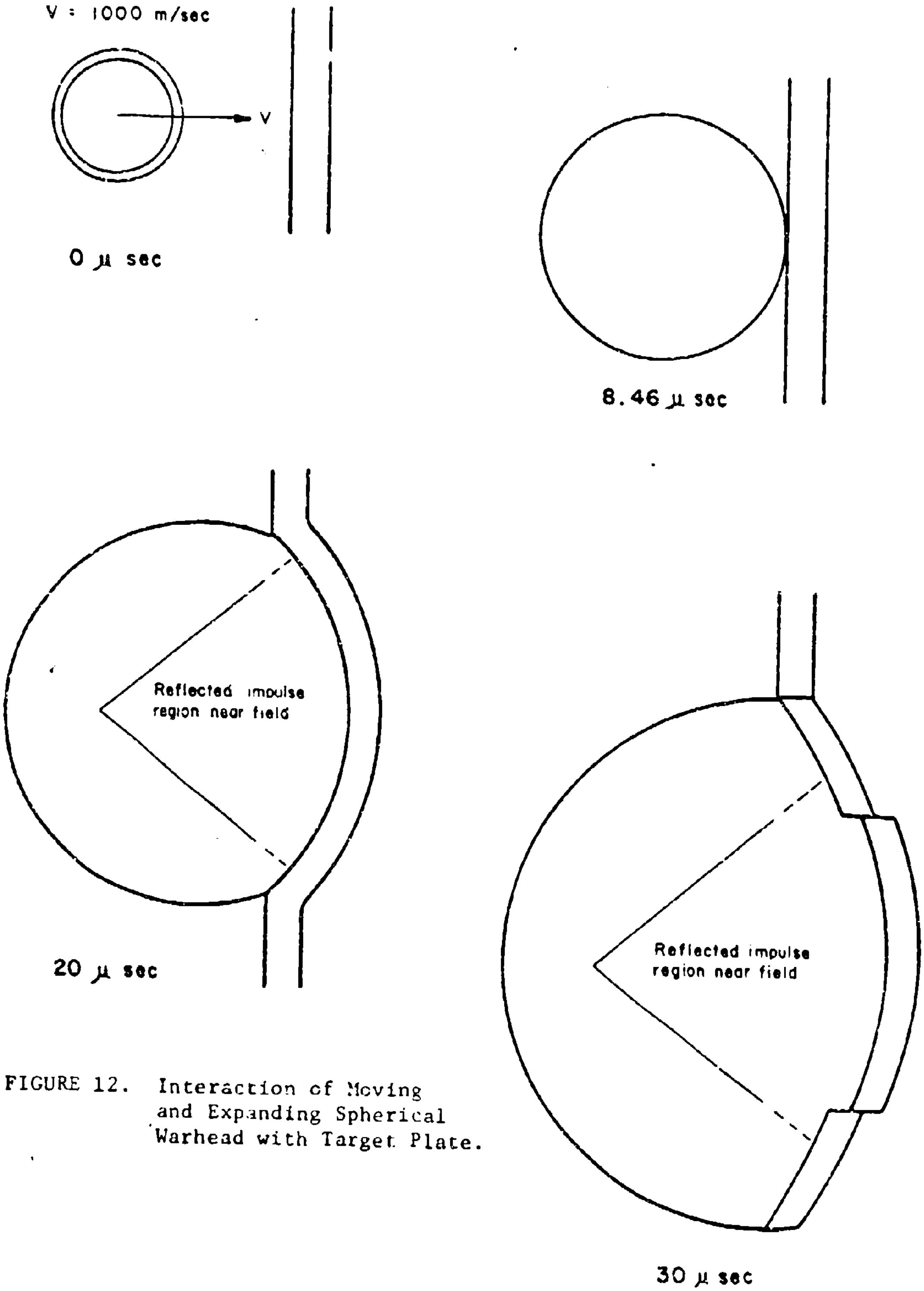

FIGURE 12. Interaction of Moving and Expanding Spherical Warhead with Target Plate.

nearly twice the expected impulse from just the fragments by the near-field effects of the detonation products.

Now let us look at an actual 20-mm HEI projectile during the early phases of its expansion as shown in Figure 13. The fragments at this stage of expansion are not the fragments observed in an arena characterization test. It is probable that the cracks that lead to the eventual separation are partially formed, but separation has not yet occurred. In the near-field zone, either a different fragmentation characterization is required, or the behavior of closely packed and aligned multiple fragments must be more thoroughly studied.

CONCLUSIONS

Blast combined with fragment momentum effects, and especially near-field blast, play an important part in the damage produced by high-explosive incendiary projectiles. The behavior of the total damage mechanism requires either a more thorough examination of the effects of multiple fragments acting in concert to produce synergistic effects, or a recharacterization of the fragmentation from HEI projectiles when operating in the near-field. Various criteria for estimating radii of effects of the various parts of the blast damage agent have been developed. These should not be used if experimental data can be obtained. The constants in the equations and criteria are theoretically derived. It is hoped that the criteria presented will provide a guide for future experimentation to determine the actual values of the constants to be used in these equations.

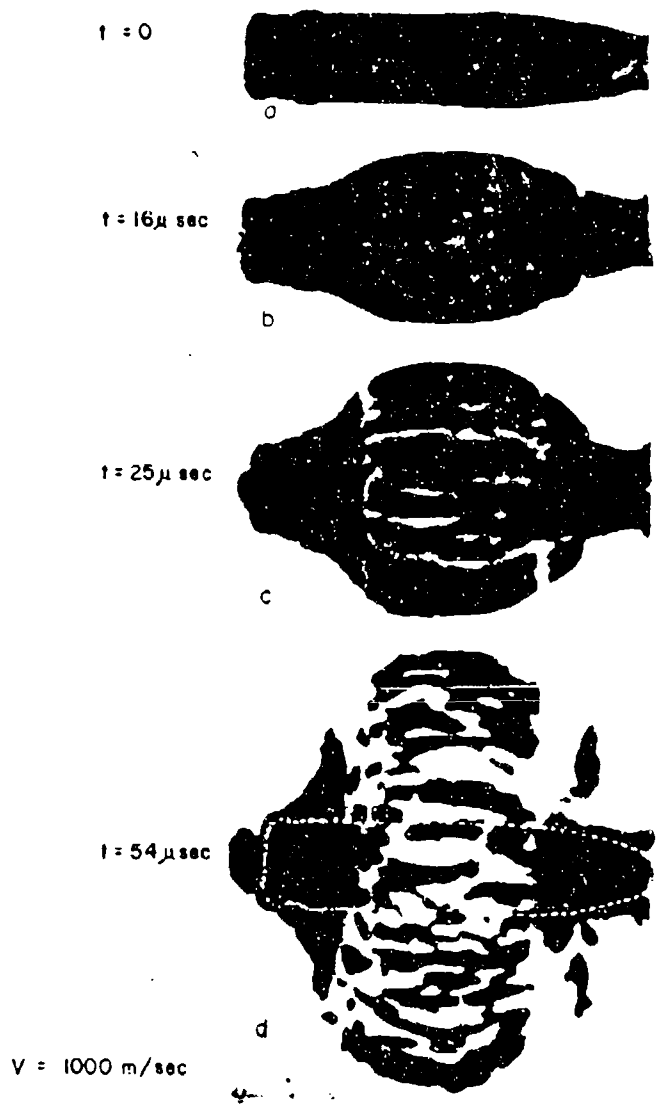

FIGURE 13. Flash Radiographs Showing the Expansion and Fragmentation of a 20-Millimeter H.E. Shell. (a) At instant detonation of high explosive reached based of shell. Relative thick shell case has started to expand: (b) approximately 16 microseconds after (a): (c) approximately 25 microseconds after (a): (d) approximately 24 microseconds after (a). The dotted line indicates undetonated shell.

NOMENCLATURE

V_o	Reference critical velocity, m/sec or ft/sec
ρ_o	Reference density, kg/m^3 or slugs/ft^3
δ_o	Reference thickness, mm or inches
V_s	Material critical velocity, m/sec or ft/sec
ρ_s	Material density, kg/m^3 or slugs/ft^3
δ_s	Material thickness, mm or inches
K_s	Not dimensionless, but depends on the units of charge mass and units of length desired
P_k	Kill probability
A_v	Vent area
S	Wall surface area, unvented
V	Compartment volume
V_f	Free volume
ρ_m	Wall mass + air mass, compartment volume
h	Altitude, in km

APPENDIX A

This appendix covers an approximate method for treating moving charge blasts. The method selected is based on the shift of the center of mass, taking momentum conservation into effect and using the mass over which the momentum is conserved as the mass encompassed by the shock wave as a function of time. Using this procedure and reference weights of explosives (W) of 1 kilogram, and reference velocity (V_c) of the explosive of 1 kilometer per second, the effective shift of the center of mass (X_c), and thus the center of the detonation as a function of time (t) are shown in Figure A-1. Since reference values were used, this becomes a universal curve which can be used with the scaling laws to enable one to shift the curve for any reasonable altitude, temperature, pressure, and explosive weight. The approach is to first calculate the effective explosive weight (W) by using equation (1).

$$W_e = W\left(1 + \frac{V_c^{\,2}}{9}\right) \qquad \text{(A-1)}$$

This equation essentially says that for an explosive moving at 3 kilometers per second, the kinetic energy is equal to the detonation energy of the explosive. The next thing that is determined is the scaled distance (Z) from the detonation point to the target in terms of the effective explosive weight (W_e), and from the tables, having entered the calculated value of W_e we determine the value of Z. Once we have determined the value of Z, we can obtain a scaled time (t_z) to the target. Now, going to the curve of shift of the center of mass (Figure A-1) with this value of t_z, we can get a scaled shift in meters of the center of mass (X_{cz}). Using the determined value of Z, we can now geometrically solve for the effective scaled distance (Z_e) to the target by means of a simple triangle solution, since we know Z, X_{cz}, and the angle theta (θ) between them. Solving this triangle (Figure A-2) gives us the value for Z_e. Knowing the value of Z_e, we can go to the tables and get the pressures (P and P_0), the scaled side-on impulse (I_{sz}), and the scaled reflected impulse (I_{RZ}). Now, through the application of the proper scaling laws, we will be able to calculate the actual overpressure (P), the scaled overpressure (P_z) to be experienced, the side-on impulse (I), and the reflected impulse (I_R).

The following equations show the relationship between the above mentioned parameters, and together with appropriate tables, permit all desired calculations to be made.

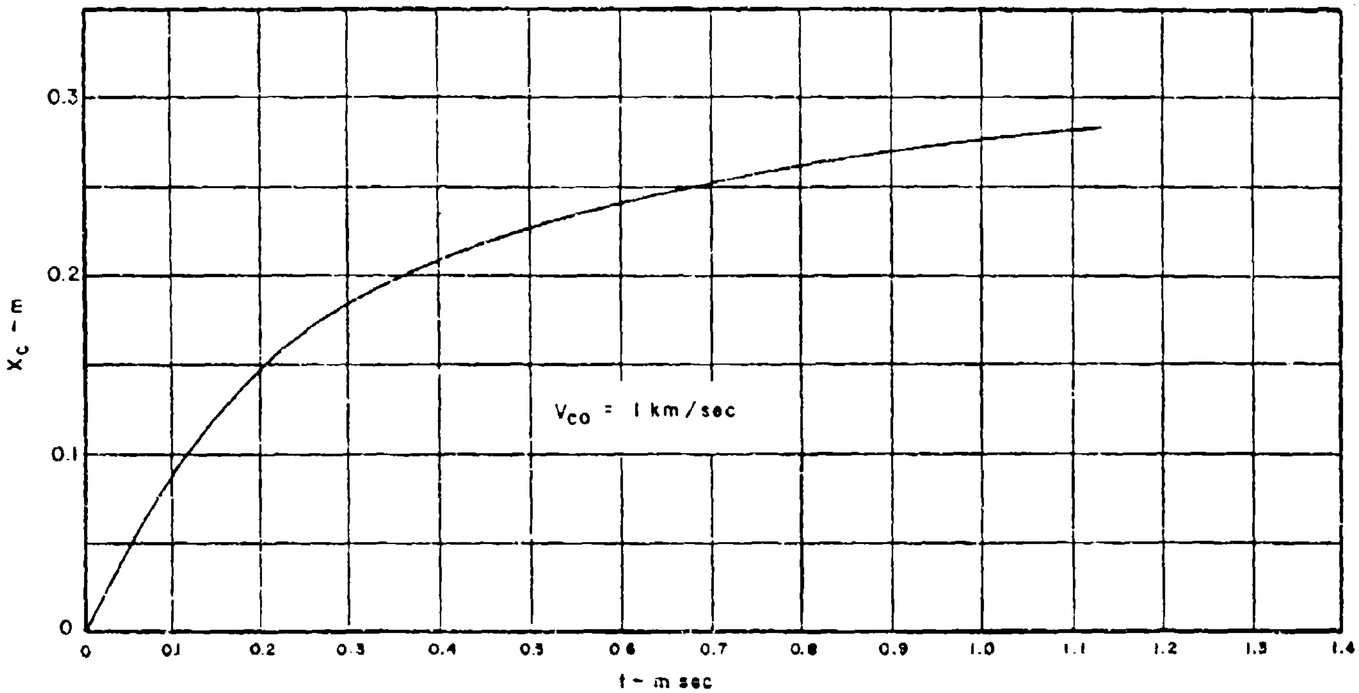

FIGURE A-1. Scaled Shift of Center of Explosion as a Function of Time for 1 kg Charge Moving at 1000 m/sec.

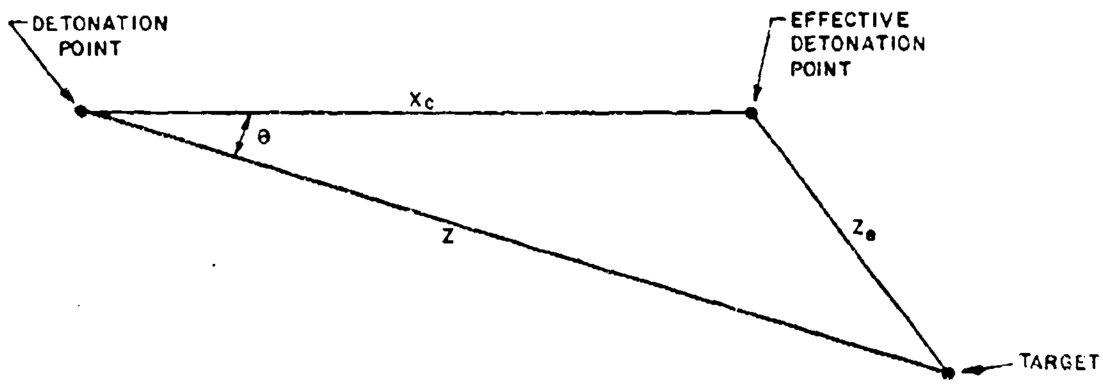

Figure A-2. Geometric Solution for Effective Scaled Distance to the Target.

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$$W_e = W(1 + \frac{V_{co}^2}{9} \qquad \text{(A-2)}$$

$$Z = \frac{R \; (^{\rho}/\rho_o)^{1/3}}{(^{W}/W_e)^{1/3}} \qquad \text{(A-3)}$$

$$t_z = \frac{t \; (^{\rho}/\rho_o)^{1/3} \; (^{a}/a_o)}{(^{W}/W_e)^{1/3}} \qquad \text{(A-4)}$$

$$t_z = \frac{t \; (^{P}/P_o)^{1/3} \; (^{T}/T_o)^{1/6}}{(^{W}/W_e)^{1/3}} \qquad \text{(A-5)}$$

$$\rho = \rho_z \; (^{P}/P_o) \qquad \text{(A-6)}$$

$$I = \frac{I_z \; (^{W}/W_e)^{1/3} \; (^{P}/P_o)^{2/3}}{(^{T}/T_o)^{1/6}} \qquad \text{(A-7)}$$

$$X_c = X_{cz} \; W_e^{1/3} \qquad \text{(A-8)}$$

NOMENCLATURE

W_e	Explosive weight
W	Effective explosive weight
V_c	Velocity in explosive, in km/sec
V_{co}	Reference velocity of explosive, in km/sec
Z	Scaled distance
Z_e	Effective scaled distance
R	Actual distance, in meters
ρ	Atmospheric density
ρ_o	Reference atmospheric density
a	Atmospheric speed of sound
a_o	Reference atmospheric speed of sound
p	Actual overpressure
p_z	Scaled overpressure
T	Atmospheric temperature, absolute
T_o	Reference atmospheric temperature, absolute
t	Actual time
t_z	Scaled time
I	Actual impulse
I_z	Scaled impulse
I_{RZ}	Scaled reflected impulse
I_s	Actual side-on impulse
I_{se}	Scaled side-on impulse
P	Atmospheric pressure
P_o	Reference atmospheric pressure
X_c	Shift in center of mass (effective center of detonation)
X_{cz}	Scaled shift of center of mass
θ	Angle between X_c and Z_e